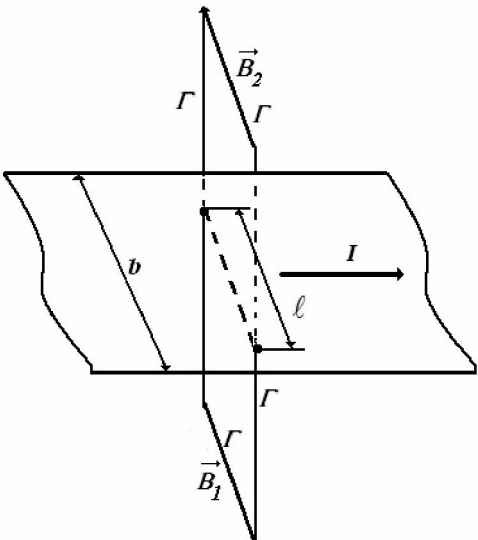
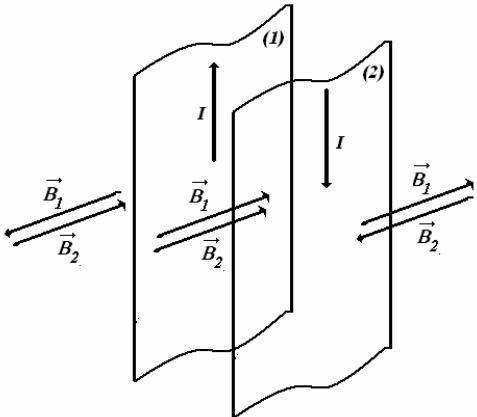


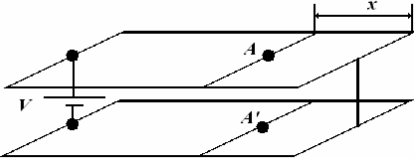
Marking scheme
Points are given for any correct solution

Problem III
Rings and strips

Nr. item	Task no. 1 – Rotating ring	Points
1.a.	For: $\vec{B}_{Earth} = B_h \cdot \hat{e}_x + B_v \cdot \hat{e}_z$ B_h - horizontal component of Earth's magnetic field 0.20p B_v - vertical component of Earth's magnetic field formula of surface vector $\vec{S}$ for the ring 0.20p $\vec{S} = \pi \cdot R^2 \cdot (\cos(\omega \cdot t) \cdot \hat{e}_x + \sin(\omega \cdot t) \cdot \hat{e}_y)$ expression of magnetic flux ϕ of Earth's magnetic field through ring's surface 0.40p $\begin{cases} \phi = \vec{B}_{Earth} \cdot \vec{S} = (B_h \cdot \hat{e}_x + B_v \cdot \hat{e}_z) \cdot (\pi \cdot R^2 \cdot (\cos(\omega \cdot t) \cdot \hat{e}_x + \sin(\omega \cdot t) \cdot \hat{e}_y)) \\ \phi = B_h \cdot \pi \cdot R^2 \cdot \cos(\omega \cdot t) \end{cases}$ expression of electromotive force induced in the ring 0.20p $\begin{cases} E = -\frac{d\phi}{dt} \\ E(t) = B_h \cdot \pi \cdot R^2 \cdot \omega \cdot \sin(\omega \cdot t) \end{cases}$ intensity of electrical current through the ring $i(t) = \frac{B_h \cdot s \cdot R \cdot \omega}{2 \cdot \rho} \cdot \sin(\omega \cdot t)$ 0.20p modulus of induced magnetic field in center of ring 0.20p $B_i(t) = \frac{\mu_0 \cdot B_h \cdot s \cdot R \cdot \omega}{4R \cdot \rho} \cdot \sin(\omega \cdot t)$ $\vec{B}_i = B_i \cdot (\hat{e}_x \cdot \cos(\omega \cdot t) + \hat{e}_y \cdot \sin(\omega \cdot t))$ 0.60p $\vec{B}_i = \frac{\mu_0 \cdot B_h \cdot s \cdot \omega}{4 \cdot \rho} \cdot \left[\hat{e}_x \cdot \frac{1}{2} \cdot \sin(2\omega \cdot t) + \hat{e}_y \cdot \frac{1}{2} \cdot (1 - \cos(2\omega \cdot t)) \right]$ average value per time of induced magnetic field $\langle \vec{B}_i \rangle = \frac{\mu_0 \cdot B_h \cdot s \cdot R \cdot \omega}{8R \cdot \rho} \cdot \hat{e}_y$ 0.20p $tg\alpha = \frac{ \langle \vec{B}_i \rangle }{B_h}$ 0.20p $\alpha = \arctg \frac{\mu_0 \cdot s \cdot \omega}{8 \cdot \rho}$ 0.20p	2.60p

Nr. item	Task no. 2 – The superconducting ring	Poins
2.a.	For: expression of magnetic flux through ring's surface $\Phi = B_z \cdot \pi \cdot r_0^2 + L \cdot I$ 0.40p $0 = R \cdot I = \frac{d\Phi}{dt}$ 0.20p - voltage drop on superconducting ring is zero - magnetic flux inside the ring is constant $\Phi = B_0 \cdot (1 - \alpha \cdot z) \cdot \pi \cdot r_0^2 + L \cdot I = \text{constant}$ Initial conditions $\begin{cases} z(t=0) = 0 \\ I(t=0) = 0 \end{cases} \quad \text{constant} = B_0 \cdot \pi \cdot r_0^2$ 0.40p expression of the intensity of electric current through the ring $I = \frac{B_0}{L} \cdot \alpha \cdot \pi \cdot r_0^2 \cdot z$ 0.40p radial component of the force of interaction is zero - because of symmetry vertical component of the force of interaction $F_z = -\frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L} \cdot z$ 0.60p elastic constant $k = \frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L}$ 0.20p equations of motion for the ring $m \cdot \frac{d^2 z}{dt^2} + k \cdot z = -m \cdot g$ 0.60p general solution of the equations of motion for the ring $z(t) = A \cdot \cos(\omega \cdot t + \psi) - \frac{m \cdot g}{k}$ 0.40p initial conditions $\begin{cases} z(0) = 0 \\ \dot{z}(0) = v_z(0) = 0 \end{cases}$ 0.20p $\begin{cases} z(t) = \frac{m \cdot g \cdot L}{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4} \cdot \cos \left[\sqrt{\frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L \cdot m}} \cdot t - 1 \right] \\ z(t) = \frac{m \cdot g \cdot L}{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4} \cdot \cos \left[\pi \cdot B_0 \cdot r_0^2 \cdot \sqrt{\frac{2 \cdot \alpha \cdot \beta}{L \cdot m}} \cdot t - 1 \right] \end{cases}$ 0.60p Observations: 1. Vertical coordinate is $z \leq 0$. 2. Electric current passes all the time in the same direction through the ring and has a minimum value ($I = 0$) in the upper point ($z = 0$) of oscillation 3. The electromagnetic force is always upward with a minimum value ($F_m = 0$) in the upper point of oscillation	4.00p

2.b.	For: expression of time dependence of electric current $I(t) = \frac{g \cdot m}{2\pi \cdot \beta \cdot B_0 \cdot r_0^2} \cdot \left(\cos \sqrt{\frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L \cdot m}} \cdot t - 1 \right)$ $I(t) = \frac{g \cdot m}{2\pi \cdot \beta \cdot B_0 \cdot r_0^2} \cdot \left(\cos \left[\pi \cdot B_0 \cdot r_0^2 \cdot \sqrt{\frac{2 \cdot \alpha \cdot \beta}{L \cdot m}} \cdot t - 1 \right] \right)$	0.40p
Nr. item	Task no. 3 – Conducting strips	Points
3.a.	For:  Ampere law $\oint_{\Gamma} \vec{B} \cdot d\vec{r} = \mu_0 \cdot \frac{I \cdot \ell}{b}$ $\begin{cases} B_1 \cdot \ell + B_2 \cdot \ell = \mu_0 \cdot \frac{I \cdot \ell}{b} \\ B_1 + B_2 = \mu_0 \cdot \frac{I}{b} \end{cases}$	0.50p
	$B_1 = B_2 = \mu_0 \cdot \frac{I}{2b}$	0.20p
3.b.	For :  $\begin{cases} B = B_1 + B_2 \\ B = \mu_0 \cdot \frac{I}{b} \end{cases}$	0.50p

3.c.	For: $\begin{cases} U = -\frac{d\varphi}{dt} \\ U = -L \cdot \frac{dI}{dt} \end{cases} \quad \text{or} \quad \varphi = L \cdot I$ 0.30p $L = \frac{\mu_0 \cdot D \cdot a}{b}$ 0.20p	0.50p
3.d.	For: the second II Kirchhoff law $V = L \cdot \frac{dI}{dt} \quad (R=0, C=0)$ 0.30p $I(t) = \frac{V \cdot b}{\mu_0 \cdot D \cdot a} \cdot t$ 0.20p	0.50p
3.e.	For:  Voltage difference $U_{AA'}$ between two mirroring points of the two bands is due to self-induced voltage corresponding to the portion starting at x to the end of the assembly strip $U_{AA'} = L(x) \cdot \frac{dI}{dt}$ 0.30p $L(x)$ - the inductance linked with of the magnetic flux through ending portion (of length x) of the strip assembly $\begin{cases} L(x) = \frac{\Phi(x)}{I} \\ L(x) = \frac{B \cdot x \cdot a}{I} \\ L(x) = \frac{\mu_0 \cdot x \cdot a}{b} \end{cases}$ $U_{AA'} = V \cdot \frac{x}{D}$ 0.20p	0.50p
3.f.	For: $\begin{cases} \frac{dW}{dt} = I \cdot U_{AA'} \\ \frac{dW}{dt} = \frac{V^2 \cdot b \cdot x}{\mu_0 \cdot D^2 \cdot a} \cdot t \end{cases}$ 0.50p	0.50p
TOTAL Problem III		10p

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