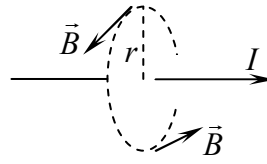


PROBLEM No. 1

a. 1p

From the symmetry of the situation we take the magnetomotive force along a circular path of radius r , centered on the wire.

$$\oint \vec{B} d\vec{l} = 2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$



b. 1.5p

Consider two elements dx of the rod placed symmetrically at distances x from its center. The corresponding forces acting on them are:

$$dF' = \frac{\mu_0 I' dx}{2\pi(d - x \sin \alpha)}$$

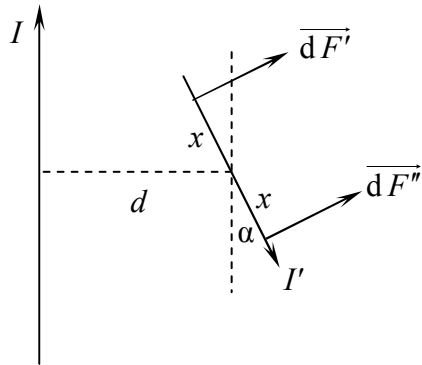
$$dF'' = \frac{\mu_0 I' dx}{2\pi(d + x \sin \alpha)}$$

The sum of their torques is:

$$dM = (dF'' - dF')x = -\frac{2\mu_0 I' x^2 \sin \alpha dx}{2\pi(d^2 - x^2 \sin^2 \alpha)}$$

For very small angles, the total torque is:

$$M = \int_0^{L/2} -\frac{\mu_0 I' \alpha x^2 dx}{\pi d^2} = -\frac{\mu_0 I' \alpha L^3}{24\pi d^2} = \frac{mL^2}{12} \ddot{\alpha} \Rightarrow T_{\text{slant}} = 2\pi d \sqrt{\frac{2\pi m}{\mu_0 I' L}}$$

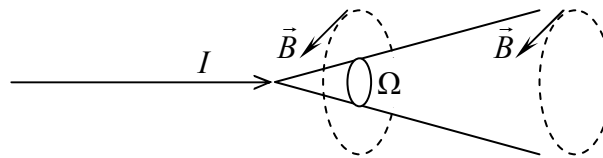


c. 1p

Taking path integrals along circular field lines exactly like at the first point, we get:

$$B_{\text{IN}} = 0$$

$$B_{\text{OUT}} = \frac{\mu_0 I}{2\pi r}$$



d. 0.5p

The above argument keeps holding, and the results are:

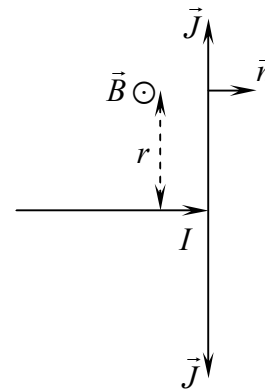
$$B_{\text{WIRE SIDE}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{OTHER SIDE}} = 0$$

e. 1p

$$J(r) = \frac{I}{2\pi r}$$

$$\Delta B_{\parallel} = B_{\parallel \text{OTHER SIDE}} - B_{\parallel \text{WIRE SIDE}} = 0 - \left(-\frac{\mu_0 I}{2\pi r}\right) = \mu_0 J$$



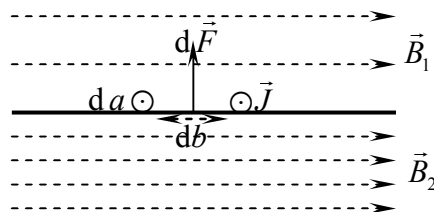
f. 1.5p

Consider a small region of the plane having dimensions da along J and db across J .

$$J = \frac{B_2 - B_1}{\mu_0}$$

Let B_0 be the external magnetic field and B' the field generated by the conducting plane.

$$\left. \begin{array}{l} B_1 = B_0 - B' \\ B_2 = B_0 + B' \end{array} \right\} \Rightarrow B_0 = \frac{B_1 + B_2}{2}$$



$$dF = dI \cdot da \cdot B_0 = J \cdot db \cdot da \frac{B_1 + B_2}{2} = \frac{B_2 - B_1}{\mu_0} dS \frac{B_1 + B_2}{2} \Rightarrow p = \frac{dF}{dS} = \frac{B_2^2 - B_1^2}{2\mu_0}$$

g. 0.5p

Just as before,

$$B_{\text{IN}} = 0$$

$$B_{\text{OUT}} = \frac{\mu_0 I}{2\pi r}$$

h. 1p

This time the path integrals go the other way around.

$$B_{\text{IN}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{OUT}} = 0$$

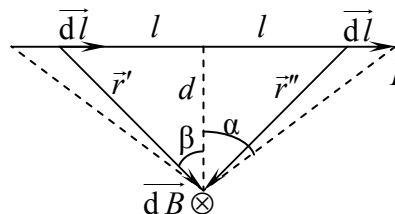
i. 1p

Consider two elements dl of the wire, placed symmetrically at a distance l from the center of the wire. Their contributions to the magnetic field in the mediator plane are equal:

$$|d\vec{B}| = \frac{\mu_0 I dl \sin(90^\circ - \beta)}{4\pi r^2} = \frac{\mu_0 I dl}{4\pi d^2} \cos^3 \beta$$

$$l = d \tan \beta \Rightarrow dl = \frac{d}{\cos^2 \beta} d\beta \Rightarrow |d\vec{B}| = \frac{\mu_0 I}{4\pi d} \cos \beta d\beta$$

$$B = 2 \int_0^\alpha \frac{\mu_0 I}{4\pi d} \cos \beta d\beta = \frac{\mu_0 I}{2\pi d} \sin \beta \Big|_0^\alpha = \frac{\mu_0 I}{\pi L} \frac{\sin^2 \alpha}{\cos \alpha}$$



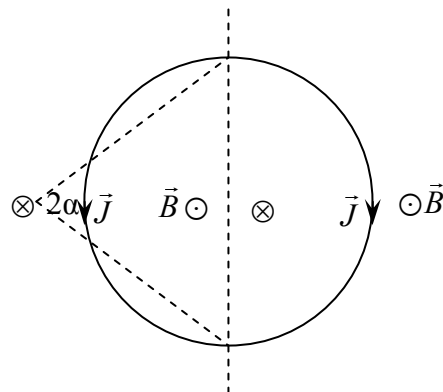
j. 1p

From a point in the equatorial plane, the axis of the poles of the sphere is seen under an angle 2α , with $\tan \alpha = R/r$.

Outside, the sphere behaves similarly to an electric current flowing directly from one pole to the other through a wire connecting the poles directly:

$$B_{\text{OUT}}(r) = \frac{\mu_0 I R}{2\pi r} \frac{1}{\sqrt{r^2 + R^2}}$$

Inside, the sphere behaves similarly to two semi-infinite straight conductors connecting the two poles of the sphere and carrying the current I in the opposite direction:



$$B_{\text{IN}}(r) = \frac{\mu_0 I}{2\pi r} \left(1 - \frac{R}{\sqrt{r^2 + R^2}} \right)$$

PROBLEM No. 2

a. 1p

Consider an element dx of the rod, placed at distance x from the center of the rod. Its mass is $dm = m dx/L$ each. Let $2l$ be the length of the rod at some moment, and let y be the corresponding length of the region x .

Since the object is homogenous at all times,

$$\frac{y}{x} = \frac{l}{L/2}$$

Let v be the velocity of the two ends of the rod at some moment.

$$v = \frac{dl}{dt} = \frac{d(2l-L)}{2dt} = \frac{d\left(\frac{2l-L}{L}\right)}{\frac{2dt}{L}} = \frac{L d\varepsilon}{2dt} = \frac{L}{2} \dot{\varepsilon}$$

The velocity of the element considered is

$$v(x) = \frac{dy}{dt} = \frac{d\left(\frac{lx}{L/2}\right)}{dt} = \frac{x}{L/2} \frac{dl}{dt} = \frac{xv}{L/2} = \frac{2x}{L} \frac{L}{2} \dot{\varepsilon} = x \dot{\varepsilon}$$

The kinetic energy of the rod is

$$E_{\text{kin}} = 2 \int_0^{L/2} \frac{dmv^2(x)}{2} = \int_0^{L/2} x^2 \dot{\varepsilon}^2 \frac{mdx}{L} = \frac{m \dot{\varepsilon}^2}{L} \frac{x^3}{3} \Big|_0^{L/2} = \frac{mL^2 \dot{\varepsilon}^2}{24}$$

b. 0.5p

Let S be the cross section of the rod, and V its volume. The elementary work done by the tensile force σS equals the increase in elastic potential energy.

$$dE_{\text{pot}} = dW = F d(2l) = \sigma S d(2l-L) = E\varepsilon \frac{V}{L} d(2l-L) = \frac{mE}{\rho} \varepsilon d\varepsilon = d\left(\frac{mE}{2\rho} \varepsilon^2\right) \Rightarrow$$

$$E_{\text{pot}} = \frac{mE \varepsilon^2}{2\rho}$$

c. 0.5p

$$E_{\text{mech}} = E_{\text{kin}} + E_{\text{pot}} = \frac{mL^2 \dot{\varepsilon}^2}{24} + \frac{mE \varepsilon^2}{2\rho} = \text{constant} \Rightarrow \dot{E}_{\text{mech}} = \frac{mL^2 \ddot{\varepsilon}}{12} + \frac{mE \varepsilon \dot{\varepsilon}}{\rho} = 0 \Rightarrow$$

$$\frac{mL^2 \ddot{\varepsilon}}{12} + V\sigma = 0$$

Dividing by L we get:

$$m \left(\frac{L\varepsilon}{12} \right)'' = -\sigma S \Rightarrow \ddot{x}_{\text{equivalent}} = \left(\frac{L\varepsilon}{12} \right)''$$

d. 0.5p

Dividing also by m we get:

$$\frac{L^2}{12} \ddot{\varepsilon} + \frac{E}{\rho} \varepsilon = 0 \Rightarrow \omega^2 = \frac{12E}{\rho L^2} \Rightarrow T_{\text{long}} = \pi L \sqrt{\frac{\rho}{3E}}$$

e. 0.5p

Consider very thin spherical layers of radius x and thickness dx . Their masses are:

$$dm = m \frac{4\pi x^2 dx}{4\pi R^3} = \frac{3m}{R^3} x^2 dx$$

Let r be the radius of the sphere and v the velocity of its surface at some moment. The argument goes similarly as in section A.

$$v = R\dot{\varepsilon} \Rightarrow v(x) = x\dot{\varepsilon} \Rightarrow dE_{\text{kin}} = \frac{dmv^2(x)}{2} = \frac{3mx^2 dx}{R^3} \frac{x^2 \dot{\varepsilon}^2}{2} \Rightarrow E_{\text{kin}} = \frac{3mR^2 \dot{\varepsilon}^2}{10}$$

$$dE_{\text{pot}} = dW = \sigma S dr = \varepsilon E 4\pi R^2 R d\varepsilon = \frac{3EV d(\varepsilon^2)}{2} = d\left(\frac{3mE}{2\rho} \varepsilon^2\right)$$

$$E_{\text{mech}} = \frac{3m}{2} \left(\frac{R^2 \dot{\varepsilon}^2}{5} + \frac{E \varepsilon^2}{\rho} \right) = \text{constant}$$

f. 0.5p

$$\dot{E} = 0 \Rightarrow \frac{R^2 \ddot{\varepsilon}}{5} + \frac{E \varepsilon \dot{\varepsilon}}{\rho} = 0 \Rightarrow \omega^2 = \frac{5E}{\rho R^2} \Rightarrow T_{\text{radial}} = 2\pi R \sqrt{\frac{\rho}{5E}}$$

g. 0.5p

$$\begin{cases} \varepsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \\ \varepsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} \end{cases} \Rightarrow \begin{cases} \sigma_x = \frac{E(\varepsilon_x + \mu \varepsilon_y)}{1 - \mu^2} \\ \sigma_y = \frac{E(\varepsilon_y + \mu \varepsilon_x)}{1 - \mu^2} \end{cases}$$

h. 0.5p

$$\begin{cases} m\ddot{x}_{\text{equivalent}} = -\sigma_x \frac{V}{L} \\ m\ddot{y}_{\text{equivalent}} = -\sigma_y \frac{V}{l} \end{cases} \Rightarrow \begin{cases} \frac{mL\ddot{x}}{12} + \frac{E(\varepsilon_x + \mu \varepsilon_y)}{1 - \mu^2} \frac{V}{L} = 0 \\ \frac{ml\ddot{y}}{12} + \frac{E(\varepsilon_y + \mu \varepsilon_x)}{1 - \mu^2} \frac{V}{l} = 0 \end{cases}$$

i. 1.5p

By replacing the sought solutions into the system of equations we get

$$\begin{cases} -\frac{\omega^2 AL^2}{12} + \frac{E(A + \mu B)}{\rho(1 - \mu^2)} = 0 \\ -\frac{\omega^2 Bl^2}{12} + \frac{E(B + \mu A)}{\rho(1 - \mu^2)} = 0 \end{cases}$$

By dividing the two equations term by term we get a simpler one:

$$\frac{AL^2}{Bl^2} = \frac{A + \mu B}{B + \mu A}$$

Let us denote the ratio of the two amplitudes by r .

$$r \frac{L^2}{l^2} = \frac{r + \mu}{1 + r\mu} \Rightarrow \mu L^2 r^2 + (L^2 - l^2)r - \mu l^2 = 0 \Rightarrow$$

$$r_{1,2} = \frac{-(L^2 - l^2) \pm \sqrt{(L^2 - l^2)^2 + 4\mu^2 L^2 l^2}}{2\mu L^2}$$

Returning r in the second equation we get:

$$\omega^2 = \frac{12E}{\rho l^2 (1 - \mu^2)} \left[1 + \mu \frac{-(L^2 - l^2) \pm \sqrt{(L^2 - l^2)^2 + 4\mu^2 L^2 l^2}}{2\mu L^2} \right] \Rightarrow$$

$$\omega_{1,2} = \sqrt{\frac{6E \left[L^2 + l^2 \pm \mu \sqrt{(L^2 - l^2)^2 + (2\mu L l)^2} \right]}{\rho L^2 l^2 (1 - \mu^2)}}$$

j. 0.5p

$$L = l \Rightarrow \omega_{1,2} = \sqrt{\frac{6E(2L^2 \pm 2\mu^2 L^2)}{\rho L^4 (1 - \mu^2)}} = \sqrt{\frac{12E(1 \pm \mu^2)}{\rho L^2 (1 - \mu^2)}}$$

$$\Delta\omega = \sqrt{\frac{12E}{\rho L^2}} \left(\sqrt{\frac{1 + \mu^2}{1 - \mu^2}} - 1 \right) \approx \mu^2 \sqrt{\frac{12E}{\rho L^2}} \Rightarrow T_{\text{beats}} = \frac{T_{\text{long}}}{\mu^2}$$

k. 1.5p

Let d be the thickness of the plate. The shear force $\tau l d$ can be decomposed into a stretching component along L (x -axis) and a shrinking component along l (y -axis).

$$\sigma_x = \frac{\tau l d \sin \gamma}{l d} ; \sigma_y = \frac{\tau l d \cos \gamma}{(L/\cos \gamma) d} \Rightarrow$$

$$\varepsilon_x = \frac{\tau \sin \gamma}{E} - \mu \left(-\frac{\tau l \cos^2 \gamma}{L E} \right)$$

$$\varepsilon_y = -\frac{\tau l \cos^2 \gamma}{L E} - \mu \frac{\tau \sin \gamma}{E}$$

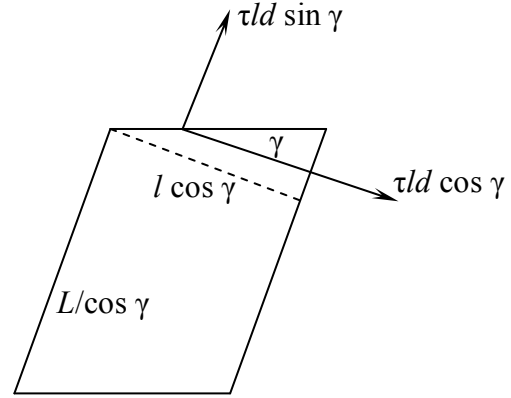
But

$$\varepsilon_x = \frac{\frac{L}{\cos \gamma} - L}{L} = \frac{1 - \cos \gamma}{\cos \gamma} ; \varepsilon_y = \frac{l \cos \gamma - l}{l} = -(1 - \cos \gamma) \Rightarrow$$

$$\begin{cases} \frac{E}{\tau} \frac{1 - \cos \gamma}{\cos \gamma} = \sin \gamma + \mu \frac{l}{L} \cos^2 \gamma \\ \frac{E}{\tau} (1 - \cos \gamma) = \frac{l}{L} \cos^2 \gamma + \mu \sin \gamma \end{cases}$$

Multiplying the second equation by μ and subtracting it from the first one we get:

$$\frac{E}{\tau} (1 - \cos \gamma) \left(\frac{1}{\cos \gamma} - \mu \right) = \sin \gamma (1 - \mu^2) \Rightarrow \frac{E \gamma^2}{2\tau} (1 - \mu) \approx \gamma (1 - \mu^2) \Rightarrow \gamma = \frac{2\tau(1 + \mu)}{E}$$



$$G = \frac{E}{2(1+\mu)}$$

l. 0.5p

The quantities involved in the shear deformation are absolutely analogous to those describing the longitudinal deformation.

$$T_{\text{slant}} = \pi L \sqrt{\frac{\rho}{3G}} = T_{\text{long}} \sqrt{2(1+\mu)}$$

m. 0.5p

Consider very thin cylindrical layers of radius x and thickness dx . When the cylinder is twisting, each one of them is subject to a very small shear.

$$T_{\text{twist}} = \pi L \sqrt{\frac{\rho}{3G}}$$

n. 1p

Let α be a very small angle with which one cap of the cylinder rotates with respect to the other. Then the slanting angle of a cylindrical layer is:

$$x\alpha = L\gamma \Rightarrow \gamma = \frac{x}{L}\alpha$$

The corresponding shear stress is

$$\tau = G \frac{x}{L} \alpha$$

The elementary shear force acting on the cap is

$$dF = \tau dS = G \frac{x}{L} \alpha 2\pi x dx$$

The corresponding elementary torque is

$$dM = dF \cdot x = \frac{2\pi G \alpha x^3 dx}{L}$$

$$M = \frac{2\pi G \alpha}{L} \int_0^R x^3 dx = \frac{2\pi G R^4 \alpha}{4L} \Rightarrow C = \frac{\pi G R^4}{2L} = \frac{\pi E R^4}{4L(1+\mu)}$$

PROBLEM No. 3

a. 0.5p

Deriving the Lorentz transformations two-fold, we get

$$a_x = a'_x \left(\frac{\sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c^2} v'_x} \right)^3$$

In our case $u = v_x$ and $v'_x = 0$.

$$\frac{dv_x}{dt} = a' \left(1 - \frac{v_x^2}{c^2} \right)^{\frac{3}{2}}$$

$$F_x = \frac{ma_x}{1 - \frac{v_x^2}{c^2}} = m_0 a' = \text{constant}$$

b. 0.5p

$$v_x = c \sin \alpha \Rightarrow \frac{d(c \sin \alpha)}{(1 - \sin^2 \alpha)^{\frac{3}{2}}} = a' dt \Rightarrow c \tan \alpha = a' t + C$$

At $t = 0$, $v_x = 0$, so $\alpha = 0$ and $C = 0$.

$$\frac{\frac{v_x}{c}}{\sqrt{1 - \frac{v_x^2}{c^2}}} = \frac{a' t}{c} \Rightarrow v = c \frac{\frac{a' t}{c}}{\sqrt{1 + \left(\frac{a' t}{c} \right)^2}}$$

c. 0.5p

$$dt' = dt \sqrt{1 - \frac{v^2}{c^2}} = \frac{dt}{\sqrt{1 + \left(\frac{a' t}{c} \right)^2}}; \frac{a' t}{c} = \sinh \tau \Rightarrow dt' = \frac{c}{a'} d\tau \Rightarrow t' = \frac{c}{a'} \tau + C$$

Again $C = 0$, so

$$t' = \frac{c}{a'} \operatorname{arcsinh} \left(\frac{a' t}{c} \right)$$

d. 1p

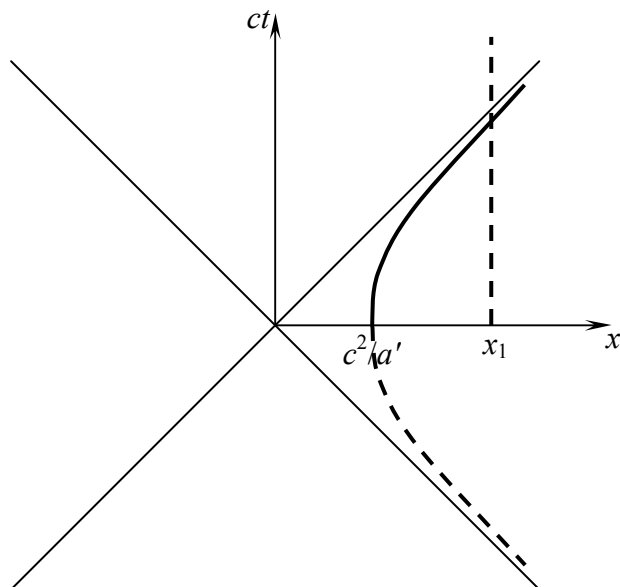
$$\left. \begin{aligned} -c^2 (dt')^2 &= -c^2 (dt)^2 + (dx)^2 \\ \frac{a' t}{c} &= \sinh \tau \Rightarrow dt = \frac{c}{a'} \cosh \tau d\tau \end{aligned} \right\} \Rightarrow (dx)^2 = \frac{c^4}{a'^2} (\cosh^2 \tau - 1) (d\tau)^2 \Rightarrow dx = \frac{c^2}{a'} \sinh \tau d\tau \Rightarrow$$

$$x = \frac{c^2}{a'} \cosh \tau + C$$

At $t = t' = 0$, $x_0 = c^2/a'$, so again $C = 0$.

e. 1p

$$ct = \frac{c^2}{a'} \sinh \tau \Rightarrow x^2 - (ct)^2 = \left(\frac{c^2}{a'}\right)^2 \Rightarrow \frac{x^2}{\left(\frac{c^2}{a'}\right)^2} - \frac{(ct)^2}{\left(\frac{c^2}{a'}\right)^2} = 1$$



f. 0.5p

$$\rho_0 = \frac{c^2}{a'} \Rightarrow \begin{cases} x = \rho_0 \cosh \tau \\ ct = \rho_0 \sinh \tau \end{cases}$$

g. 0.5p

$$\begin{cases} x = \rho \cosh \tau \\ ct = \rho \sinh \tau \end{cases} ; \begin{cases} \rho = \sqrt{x^2 - (ct)^2} \\ \tau = \operatorname{arctanh}\left(\frac{ct}{x}\right) \end{cases}$$

These equations require that $x > 0$ and $\rho > 0$, so using these new parameters one can cover only the quadrant of spacetime characterized by $x > |ct|$.

h. 1p

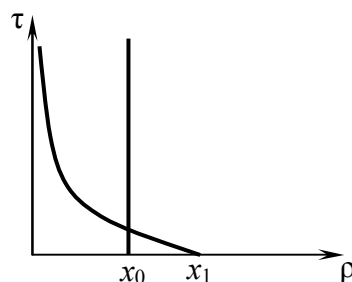
$$\left. \begin{aligned} dx &= d\rho \cosh \tau + \rho \sinh \tau d\tau \\ d(ct) &= c dt = d\rho \sinh \tau + \rho \cosh \tau d\tau \end{aligned} \right\} \Rightarrow$$

$$ds^2 = -c^2 (dt)^2 + (dx)^2 = (d\rho)^2 - \rho^2 (d\tau)^2 = -c^2 \frac{\rho^2}{c^2} (d\tau)^2 + (d\rho)^2 ; f = \frac{\rho^2}{c^2} ; g = 1$$

i. 0.5p

$$\rho = \frac{x_1}{\cosh \tau} \Leftrightarrow \tau = \operatorname{arccosh}\left(\frac{x_1}{\rho}\right)$$

$$\Delta\rho = \frac{c^2}{a'}$$



j. 0.5p

The observer will receive only those signals emitted before the beacon exits the quadrant of spacetime described by the Rindler metric.

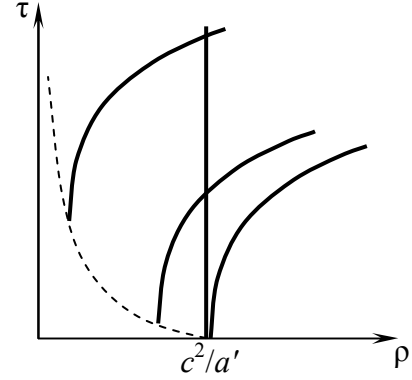
$$ct \leq x = x_0 \Rightarrow t_{\text{lim}} = \frac{x_0}{c} \Rightarrow N = \left\lfloor \frac{t_{\text{lim}}}{T_0} \right\rfloor + 1 = \left\lfloor \frac{x_0}{cT_0} \right\rfloor + 1$$

In the case of the light,

$$ds^2 = 0 \Rightarrow (d\rho)^2 = \rho^2 (d\tau)^2 \Rightarrow \frac{d\rho}{\rho} = d\tau$$

At $\tau = 0$, $\rho_0 = x_0$, so

$$\ln \frac{\rho}{\rho_0} = \tau \Rightarrow \rho = x_0 e^\tau$$



k. 1.5p

Let ρ_e and τ_e be the spacetime coordinates for the emission of a pulse.

$$\rho_e = \sqrt{x_0^2 - c^2 t^2} ; \tau_e = \text{arctanh} \left(\frac{ct}{x_0} \right)$$

$$\tanh \tau_e = \frac{e^{\tau_e} - e^{-\tau_e}}{e^{\tau_e} + e^{-\tau_e}} = \frac{ct}{x_0} \Rightarrow e^{2\tau_e} - 1 = \frac{ct}{x_0} (e^{\tau_e} + 1) \Rightarrow e^{2\tau_e} = \frac{1 + \frac{ct}{x_0}}{1 - \frac{ct}{x_0}} \Rightarrow e^{\tau_e} = \sqrt{\frac{x_0 + ct}{x_0 - ct}}$$

$$\rho = \frac{\rho_e}{e^{\tau_e}} e^\tau = (x_0 - ct) e^\tau = \rho_0 \Rightarrow e^\tau = \frac{x_0}{x_0 - ct} \Rightarrow \tau = \ln \left(\frac{x_0}{x_0 - ct} \right)$$

Let t^* be the moment the observer receives the last signal.

$$v(t^*) = c \frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c} \right)^2}}$$

The frequency received is

$$\nu = \nu_0 \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \nu_0 \sqrt{\frac{1 - \frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c} \right)^2}}}{1 + \frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c} \right)^2}}}} = \nu_0 \sqrt{\frac{\sqrt{1 + \left(\frac{a't^*}{c} \right)^2} - \frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c} \right)^2} + \frac{a't^*}{c}}} = \nu_0 \left[\sqrt{1 + \left(\frac{a't^*}{c} \right)^2} - \frac{a't^*}{c} \right]$$

But

$$ct^* = \frac{c^2}{a'} \sinh \tau \Rightarrow \frac{a't^*}{c} = \sinh \tau \Rightarrow \nu = \nu_0 (\cosh \tau - \sinh \tau) = \nu_0 e^{-\tau} = \nu_0 \left(1 - \frac{ct}{x_0} \right)$$

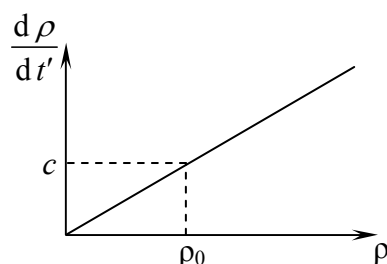
$$t = NT_0 \Rightarrow \nu = \nu_0 \left\{ 1 - \frac{cT_0}{x_0} \left(\left[\frac{x_0}{cT_0} \right] + 1 \right) \right\}$$

l. 0.5p

$$\frac{d\rho}{dt'} = \frac{d\rho}{d\tau} \frac{d\tau}{dt'} = (x_0 - ct) e^\tau \frac{a'}{c} = \frac{a'}{c} \rho$$

Upon reception,

$$e^\tau = \frac{x_0}{x_0 - ct} \Rightarrow \frac{d\rho}{dt'} = (x_0 - ct) \frac{x_0}{x_0 - ct} \frac{a'}{c} = \frac{c^2}{a'} \frac{a'}{c} = c$$



m. 1p

$$\tanh \tau = \frac{ct}{x_0} \Rightarrow \frac{1}{\cosh^2 \tau} d\tau = \frac{c}{x_0} dt \Rightarrow dt = \frac{x_0}{c} (1 - \tanh^2 \tau) d\tau = \frac{x_0}{c} \left(1 - \frac{c^2 t^2}{x_0^2} \right) d\tau$$

$$\frac{d(dt)}{dx_0} = \frac{d\tau}{c} \frac{d\left(\frac{x_0^2 - c^2 t^2}{x_0} \right)}{dx_0} = \frac{d\tau}{c} \frac{x_0^2 + c^2 t^2}{x_0^2}$$

$$\varepsilon = \frac{\frac{d\tau}{c} \frac{x_0^2 + c^2 t^2}{x_0^2} \Delta x_0}{\frac{d\tau}{c} \frac{x_0^2 - c^2 t^2}{x_0}} = \frac{x_0^2 + c^2 t^2}{x_0^2 - c^2 t^2} \frac{\Delta x_0}{x_0}$$

n. 0.5p

$$\varepsilon = \frac{\Delta x_0}{x_0} = \frac{h}{\frac{c^2}{a'}} = \frac{gh}{c^2} \approx \frac{10 \text{ m/s}^2 \cdot 360 \cdot 10^3 \text{ km}}{9 \cdot 10^{16} \text{ m}^2/\text{s}^2} = 4 \cdot 10^{-11}$$

$$\Delta t = 4 \cdot 10^{-11} \cdot 365 \cdot 24 \cdot 3600 \approx 1.26 \cdot 10^{-3} \text{ s}$$