# 4 INPhO-2009

(39).

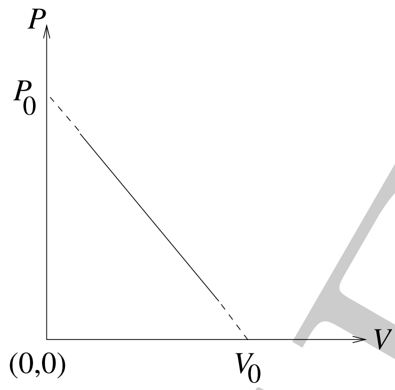


Figure 24: Problem 39

(a) Since the P-V plot is linear (see Fig. (24))

$$P = mV + C$$

Note that at $P = 0$ , $V = V_0$. Hence

$$C = -mV_0$$

$$m = \frac{-P_0}{V_0}$$

$$P = \frac{-P_0}{V_0}(V - V_0) \tag{54}$$

Rewriting we obtain

$$\frac{P}{P_0} + \frac{V}{V_0} = 1 \qquad (P < P_0, V < V_0) \tag{55}$$

(b) The ideal gas law ($PV = RT$) for one mole of the gas implies

$$P = \frac{RT}{V}$$

Using Eq.(55),

$$\frac{RT}{V} = -\frac{P_0}{V_0}(V - V_0)$$

Rewriting,

$$T = \frac{P_0 V}{R}\left(1 - \frac{V}{V_0}\right) \tag{56}$$

(c) From part (b)

$$\frac{RT}{P_0} = V\left(1 - \frac{V}{V_0}\right)$$

Differentiating with respect to T,

$$\implies \frac{R}{P_0} = \frac{dV}{dT}\left(1 - \frac{V}{V_0}\right) + V\left(-\frac{1}{V_0}\frac{dV}{dT}\right)$$

$$= \frac{dV}{dT}\left(1 - \frac{2V}{V_0}\right)$$

Rewriting,

$$\frac{dV}{dT} = \frac{RV_0}{P_0(V_0 - 2V)} \tag{57}$$

(d) From part (c)

$$\frac{dT}{dV} = \frac{P_0}{R}\left(1 - \frac{2V}{V_0}\right)$$

For maximum temperature,

$$\frac{dT}{dV} = 0$$

This occurs at

$$V = V_0/2$$

Hence

$$T_{max} = \frac{P_0 V_0}{4R} \tag{58}$$

(e)

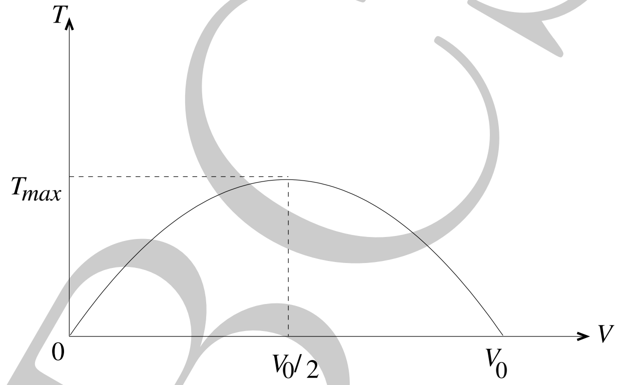


Figure 25: Problem 39 (e)

Eq. (56) is a quadratic in $V$. See Fig. (25)

(f) Note that

$$C_p - C_v = R$$

and

$$C_v = C_p/\gamma$$

Hence

$$C_v = \frac{R}{\gamma - 1}$$

(g) From the first law of thermodynamics

$$\triangle Q = \triangle U + \triangle W$$

where

$$\triangle W = PdV$$
$$\triangle Q = CdT$$

$$\triangle U = C_v dT$$

Rewriting the first law,

$$CdT = C_v dT + PdV$$

$$\therefore C = C_v + P\frac{dV}{dT}$$

$$= \frac{R}{\gamma - 1} + P_0\left(1 - \frac{V}{V_0}\right)\frac{R}{P_0\left(1 - \frac{2V}{V_0}\right)} \quad \text{(From Eq. (54) and (57))}$$

Rewriting,

$$C = \frac{R}{\gamma - 1} + \frac{(V_0 - V)R}{(V_0 - 2V)} \tag{59}$$

(h) For the mixture of gases the "adiabatic" constant is

$$\gamma = \frac{\sum n_i C_{pi}}{\sum n_i C_{vi}}$$

$$= \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R} \quad \text{(Since } n = 1/2\text{)}$$

$$= \frac{3}{2}$$

(i) Using $\gamma = \frac{3}{2}$ in Eq. (59) we obtain

$$C = R\frac{\left(3 - \frac{5V}{V_0}\right)}{\left(1 - \frac{2V}{V_0}\right)}$$

(j) Note that negative specific heat implies that temperature goes down although heat is being pumped into the system. This is on account of the peculiar linear nature of the process. Also note that temperature too decreases for $V > V_0/2$, the students are advised to calculate.

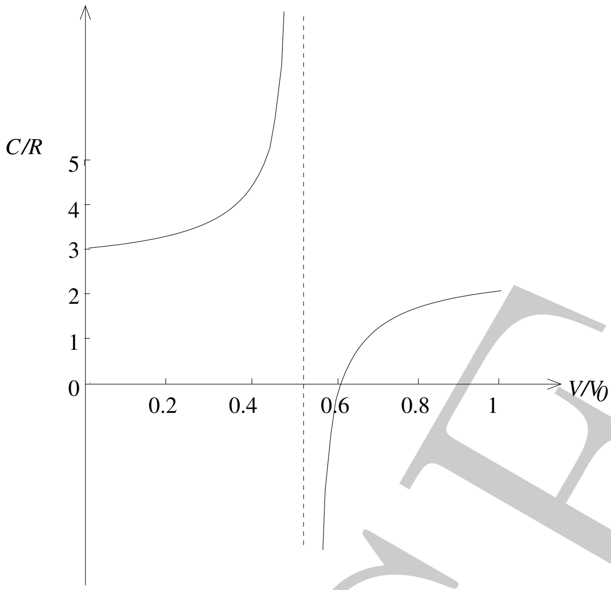


Figure 26: Problem 39 (j)

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(29 July 1898-11 Jan.1988)

**"Some of the young people I see, who are very good, take physics... as a system you can do things with, can calculate something with, and they miss... the mystery of it : how very different it is from what you can see, and how profound nature is...There is no good translation of the Yiddish word '*Witz*'. It's a joke or a trick or a sleight of hand. You can always bulldoze your way to an answer, but it's the use of this kind of witty trick or subtle approach that I have always liked about physics.... I have always taken physics personally....It's been me and nature and nature is sacred and profound."**

-Isidor Isaac Rabi, Nobel Laureate, 1944 on how not to approach a physics problem and how to view nature with respect and awe.

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