PN junctions in semiconductors

Semiconductors are materials with electric conductivity significantly lower than conductivity of metals. Unlike insulators, conductivity of semiconductors is not zero and rapidly increases with temperature. This fact can be explained by the increase of charge carrier concentration due to thermal excitations.

There are two types of charge carriers in semiconductors, electrons and holes. Holes are positively charged particles, that appear in place where an electron is absent. Hole charge is equal to absolute value of electron charge. In a pure semiconductor concentrations of electrons and holes are equal to each other.

Semiconductor devices, in particular diodes and transistors, are widely used in modern electronics. In this problem we consider one of the simplest models of a semiconductor diode.

Charge carriers concentration in semiconductors can be easily modified by adding a small amount of impurities, i.e., atoms of another chemical element. Impurities that increase the electron concentration are referred to as donors, semiconductor with excess of donors is called n-type (from negative) semiconductor. Hole concentration in n-type semiconductor is much lower than electron concentration. Impurities that increase holes concentration are referred to as acceptors, semiconductor with excess of acceptors is called p-type (from positive) semiconductor.

Consider a contact of two semiconductors of p and n types (pn junction). In practice we can dope different parts of semiconductor with different types of impurities. The current through the junction depends on the polarity of applied voltage. The resulting structure is a semiconductor diode.

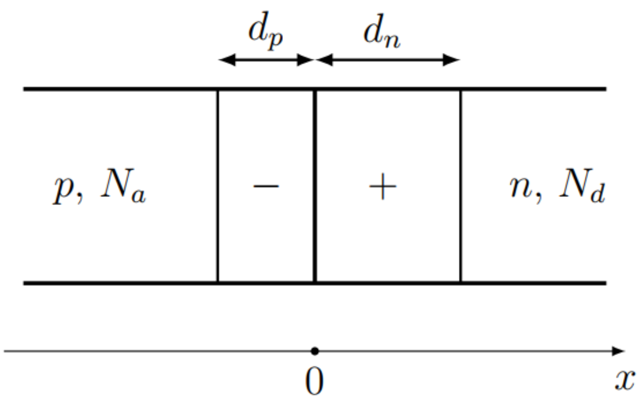

Fig. 1. Junction and depletion layers

In this problem we consider the following model of a diode (Fig. 1).

1. All physical quantities depend only on one coordinate x.
2. The region $x < 0$ is a p-type semiconductor with impurity concentration N_a.
3. The region $x > 0$ is a n-type semiconductor with impurity concentration N_d.
4. Near the junction electrons and holes diffuse through the boundary. As a result, in region $x < 0$ there is a negatively charged layer of width d_p, and in region $x > 0$ there is a positively charged layer of width d_n. These layers are called depletion layers.
5. In depletion layers the main charge carrier concentration (holes in p-type semiconductors and electrons in n-type semiconductor) is almost zero, outside of the layers their concentration is constant and equal to the concentration of impurities.

6. Volume charge density in depletion layer can be calculated from assumption that each donor impurity has charge $+q_e$, and each acceptor impurity has charge $-q_e$, where q_e – absolute value of electron charge.

You may use the following physical constants

- $\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{F/m}$ – electric constant;
- $q_e = 1.60 \cdot 10^{-19}\ \mathrm{C}$ – absolute value of electron charge;
- $k = 1.38 \cdot 10^{-23}\ \mathrm{J/K}$ – Boltzmann constant;

For the silicon based diode, considered in this problem:

- $\varepsilon = 11.7$ – dielectric constant;
- $T = 300\ \mathrm{K}$ – temperature;
- $N_a = 1.0 \cdot 10^{23}\ \mathrm{m^{-3}}$;
- $N_d = 5.0 \cdot 10^{22}\ \mathrm{m^{-3}}$;
- $n_i = 1.0 \cdot 10^{16}\ \mathrm{m^{-3}}$ – concentration of electrons and holes in semiconductor without impurities.

Part A. Electrostatics (5.0 points)

In this part we consider electric field distribution in the junction without external voltage. From symmetry considerations, the electric field is parallel to the x axis, the only non-zero component is E_x.

A.1	Total electric charge near the junction must be equal to zero. Using this fact, write an equation connecting widths of depletion layers d_n, d_p and impurities concentrations N_a, N_d.	0.5pt

A.2	Show that the following equation holds for electric field $$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_0 \varepsilon},$$ where ρ – volume charge density in the corresponding point.	0.5pt

A.3	Find the electric field $E_x(x)$ as a function of x. Assume that far from junction the field is equal to 0. Express the answer in terms of N_a, N_d, d_n, d_p, ε, ε_0 and q_e.	1.0pt

A.4	Find the electrostatic potential $\varphi(x)$ as a function of x. Assume that $\varphi(0) = 0$. Express the answer in terms of N_a, N_d, d_n, d_p, ε, ε_0 and q_e. Determine the potential difference at large distance to the junction $\Delta\varphi = \varphi(x = +\infty) - \varphi(x = -\infty)$.	1.0pt

A.5	Draw a qualitative plot of the potential $\varphi(x)$ as function of x.	0.5pt

A.6	Actually, the potential difference is determined by the material properties and the temperature. Knowing the potential difference, one can calculate depletion layers widths. Express the widths d_n, d_p in terms of $\Delta\varphi$, N_a, N_d, ε, ε_0 and q_e. Calculate their numerical values for $\Delta\varphi = 0.8\ \mathrm{V}$.	1.0pt

A.7	Determine the maximum value of the electric field in the junction $E_{\max}$. Express the answer in terms of $\Delta\varphi$, N_a, N_d, ε, ε_0 and q_e. Calculate its numerical value for $\Delta\varphi = 0.8\ \mathrm{V}$.	0.5pt

Part B. Junction with external voltage (5.0 points)

Now we apply voltage V to the junction (Fig. 2). The voltage is positive if applied to the p-type semiconductor. The applied voltage changes energies of electrons and holes in the p-type semiconductor, while energies in n-type semiconductor are the same. Assume that electrons and holes concentrations $n(x)$ and $p(x)$ obey the Boltzmann distribution:

$$n(x) = n_0 \exp\left(-\frac{U_n(x)}{kT}\right), \quad p(x) = p_0 \exp\left(-\frac{U_p(x)}{kT}\right),$$

where $U_n(x)$, $U_p(x)$ are potential energies of an electron or a hole in corresponding point, T – absolute temperature, $n_0,\ p_0$ – normalization constants, determined by the concentration of impurities N_d and N_a correspondingly. Concentration ratio of electrons and holes to the right and to the left of the junction is determined by energy difference:

$$\frac{n(x=+\infty)}{n(x=-\infty)} = \exp\left(-\frac{U_n(x=+\infty) - U_n(x=-\infty)}{kT}\right); \quad \frac{p(x=+\infty)}{p(x=-\infty)} = \exp\left(-\frac{U_p(x=+\infty) - U_p(x=-\infty)}{kT}\right).$$

If applied voltage $V > 0$, holes move form p-type semiconductor to n-type, while electrons move from n-type semiconductor to p-type. In addition, concentration of electrons in n-type semiconductor is determined by impurity concentration and does not change, while hole concentration increases. Similarly in p-type semiconductor hole concentration does not change while electron concentration increases.

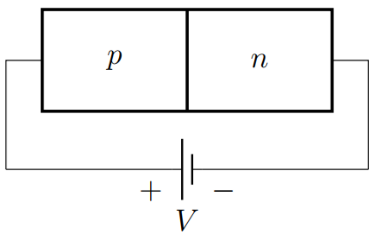

Fig. 2. Voltage applied to pn junction

B.1	Let n_{10} denote the concentration of electrons in p-type semiconductor without external voltage. Similarly let p_{10} denote the concentration of holes in n-type semiconductor without external voltage. Determine electron concentration n_1 in p-type semiconductor and hole concentration p_1 in n-type semiconductor after the voltage V is applied. Consider region outside of depletion layers, where electric field is zero. Express the answers in terms of p_{10}, n_{10}, V, T, k and q_e.	0.5pt

We obtained, that after the voltage is applied, the hole concentration in n-type semiconductor and electron concentration in p-type semiconductor are different from equilibrium ones. Electrons can move to the vacant places, corresponding to holes. In such process the electron and the hole disappear (recombine). Boltzmann distribution does not take recombination into account. Therefore without diffusion charge carries concentrations change to equilibrium values.

To be specific, consider holes in n-type semiconductor outside of depletion layer. Denote the hole concentration by p_1, equilibrium concentration without diffusion by p_{10}. The rate of concentration change due to recombination can be expressed as

$$\left(\frac{dp_1}{dt}\right)_{\mathrm{rec}} = -\frac{p_1 - p_{10}}{\tau_p},$$

where τ_p is time of hole recombination. Holes flux (number of holes that transfer through the unit area in unit time) due to diffusion is described by the formula

$$J_p = -D_p \frac{dp_1}{dx},$$

where D_p is holes diffusion coefficient.

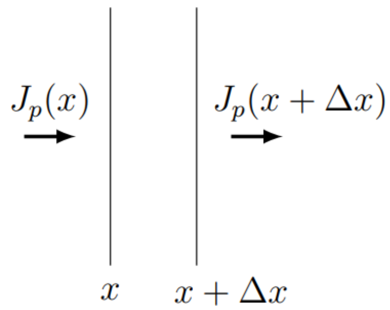

Fig. 3. Holes balance

B.2	Consider the volume between the planes with coordinates x and $x + \Delta x$ (Fig. 3). Write down the condition that the holes concentration in this volume does not change. Take into account the holes flux through the two planes and holes recombination. In the limit $\Delta x \to 0$ obtain a differential equation for the concentration deviation from equilibrium value $\Delta p = p_1 - p_{10}$ as a function of x. The answer may also include D_p and τ_p.	1.0pt

B.3	Find the distance L_p, over which the deviation of holes concentration from equilibrium decreases by e times. Express the answer in terms of D_p, τ_p. Compute numerical value for $D_p = 1.0 \cdot 10^{-3}\ \mathrm{m^2/s}$, $\tau_p = 5.0 \cdot 10^{-7}\ \mathrm{s}$.	0.6pt

The characteristic length, over which the hole concentration changes due to recombination, is much larger than the depletion layer width. Therefore we can neglect recombination in depletion layer. The current through the junction can be calculated as a sum of electron current and hole current, independent of each other. To calculate the hole current we can consider n-type semiconductor near the depletion layer boundary, where the current is determined by diffusion and hole concentration can be calculated from Boltzmann distribution. The electron current can be found in the same way.

B.4	Determine the hole concentration $p_1(x)$ in region $x > d_n$ as a function of coordinate (in n-type semiconductor outside of the depletion layer). Express the answer in terms of p_{10}, V, T, L_p, d_n, k and q_e.	0.8pt

B.5	Determine the density of hole diffusion current j_p at the boundary of depletion layer $x = d_n$. Express the answer in terms of p_{10}, V, L_p, D_p, T, k and q_e.	0.8pt

The expression for the electron current can be obtained by analogy, if we replace hole concentration by electron concentration n_{10}, diffusion coefficient, recombination time and characteristic length by corresponding values for electrons, D_n, τ_n and L_n. Equilibrium holes and electrons concentrations are given by

$$p_{10} = \frac{n_i^2}{N_d}, \quad n_{10} = \frac{n_i^2}{N_a},$$

where n_i is concentration of electrons or holes in semiconductor without impurities.

B.6	The junction cross section area is A. Derive the expression for the total current I through the junction as a function of applied voltage V, semiconductor parameters N_a, N_d, τ_p, τ_n, D_p, D_n, n_i and constants k, q_e.	0.8pt

B.7	Draw a qualitative plot of the relation $I(V)$, obtained in previous task.	0.3pt

The numerical values of the parameters are

- $\tau_n = \tau_p = 5.0 \cdot 10^{-7}$ s;
- $D_n = 2.5 \cdot 10^{-3}\ \mathrm{m^2/s}, \quad D_p = 1.0 \cdot 10^{-3}\ \mathrm{m^2/s}$;
- $A = 1.0 \cdot 10^{-8}\ \mathrm{m^2}$.

B.8	Calculate the numerical values of the saturation current I_∞ at large negative voltage, and the value of current I_1 for $V = 0.7$ V.	0.2pt