

T1. Stopping power

In this task we describe one of the processes responsible for the stopping of charged particles in a medium in the limit of low energies. The media consists of atoms with the atomic number Z_0 and concentration n_0 .

During the problem we consider relativistic effect, so we remind you that the transition to the frame moving with the velocity u along the axis x (boost along x) is accompanied by the coordinate transition corresponding to the rotation in Minkowski space:

$$\begin{cases} ct' = ct \cosh \theta - x \sinh \theta \\ x' = x \cosh \theta - ct \sinh \theta \\ y' = y \\ z' = z \end{cases},$$

where $\sinh \theta = \gamma_u u/c$, $\cosh \theta = \gamma_u$ and $\gamma_u = 1/\sqrt{1 - u^2/c^2}$.

Let the muon (a particle of mass M and charge $-e$) move with a high velocity v in the medium, where $\gamma = 1/\sqrt{1 - v^2/c^2} \ll M/m$. Let us assume that it interacts with some particle of mass m and charge Ze , passing it at a distance b **without any significant trajectory deviation or change in velocity**. In the lab frame (LF) the particle is initially at rest at the origin, and the coordinates of the muon $x = vt$, $y = b$, $z = 0$. Let's consider the frame of the muon (MF).

A1	Determine the coordinates x'_1, y'_1 of the muon in MF as the function of time t in LF. Determine the coordinates x'_2, y'_2 of the particle in MF as the function of time t in LF. Express the answers in terms of γ, v and t .
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A2	Determine the electric $\vec{E}'$ and magnetic $\vec{B}'$ fields created by the muon in the MF at the point where the particle is located. Express the answer in terms of $e, \varepsilon_0, b, \gamma, v$ and t .
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A boost with velocity u along the x -axis also transforms the EM field according to

$$\begin{cases} E'_x = E_x \\ B'_x = B_x \\ \vec{E}'_{\perp} = \gamma_u \left(\vec{E}_{\perp} + u \hat{x} \times \vec{B} \right) \\ \vec{B}'_{\perp} = \gamma_u \left(\vec{B}_{\perp} - \frac{u}{c^2} \hat{x} \times \vec{E} \right) \end{cases},$$

where the subscript $\perp$ denotes the component of the field that is perpendicular to the x -axis and $\hat{x}$ is the unit vector of the x -axis.

A3	Determine the electric $\vec{E}$ and magnetic $\vec{B}$ fields created in the LF by the muon at the point where the particle is located. Express the answer in terms of $e, \varepsilon_0, b, \gamma, v$ and t .
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The particle begins to move due to the interaction with the electric field of the muon. Assume that the particle remains classical, i.e. the momentum acquired from the interaction with the muon, Δp , satisfies $\Delta p \ll mc$.

A4	Express $ \Delta p $ in terms of $Z, e, \varepsilon_0, b, v$ and
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$$I = \int_{-\infty}^{+\infty} \frac{dx}{(1 + x^2)^{3/2}}.$$

Assume that the particle doesn't have enough time to move during the interaction with the muon. Along which axis is the momentum of the particle changed?

To calculate the integral I , the hyperbolic substitution $x = \sinh u$ may be useful.

A5	Express $ \Delta p $ in terms of $Z, e, \varepsilon_0, b, v$.
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If, due to the interaction with the muon, the particle acquires the momentum $|\Delta p|$, it also acquires energy $\Delta E = |\Delta p|^2/(2m)$ from the muon, corresponding to the muon's energy loss.

A6	Express ΔE in terms of $Z, e, \varepsilon_0, b, v, m$.
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Considering that the ratio of the proton mass to the electron mass is $m_p/m_e = 1836$ and in a solid material, the number of electrons is much greater, we assume that only electrons contribute to the stopping of the muon. Also, the process described in question A6 occurs only for $b_{\min} < b < b_{\max}$.

A7	Determine the specific energy loss per unit length L of the medium, dE/dL , for the muon. Express the answer in terms of $Z_0, n_0, e, \varepsilon_0, v, m_e, b_{\max}/b_{\min}$.
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The upper cutoff $b_{\max}$ arises because the interaction between the electron and the muon becomes too slow when the latter passes too far away. In such a case, it interacts not with a single electron but with the entire atom: $b_{\max} \approx \gamma v/\omega_0$, where ω_0 is the angular frequency of the electron's orbit in the atom.

The lower cutoff $b_{\min}$ arises due to the quantum effects and equals to the de Broglie wavelength $\hbar/(\gamma m_e v)$ of the electron in the MF.

T2. Birks’ law

When particles moves through the scintillator, it loses energy, which is partially transformed into the photons that the detector collects. Let us call the response of the detector L , the energy of the particle E and the distance traveled inside the scintillator x . Thus,

$$dL = S(E) \cdot dE,$$

where $S(E)$ characterizes the material of the characterizes and the circuit of the photon detection. The dependence of S on E arises from changes in the matter when strong interaction with the particle (high $|dE/dx|$) takes place.

Therefore, if a particle of energy E_0 hits the detector, the response is

$$L = \int_0^{E_0} S(E) dE.$$

John B. Briks suggested an empirical law to describe how S depends on the E :

$$S(E) = \frac{S_0}{1 + kB \cdot \frac{dE}{dx}},$$

where S_0 is a constant that can be measured for particles that weakly interact with the matter. The dependence of S on E is not explicit and is hidden in the fact that dE/dx is a function of E . The constant kB characterizes the internal dynamics of the matter and varies weakly with the type of particles.

The dependence of the response L of the detector made of anthracene on the energy E_0 of the proton incident on the detector is provided. Also, the stopping power dE/dx of the protons in anthracene as the function of their energy E_0 is given. Both tables can be found in *Extras* on pho.rs.

A1	Determine the value of kB for anthracene.
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