16 August 2022
IOAA
Georgia

General Instructions

Cover Sheet

- **Do not touch envelopes until the start of the examination.**
- The theoretical examination lasts for 5 hours and is worth a total of 300 marks.
- There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in
    - Student Code (Country Code and student's number. **example: GEO-3**)
    - Question Number
    - Page Number and total number of pages for **this Question**.
- ***Start each problem on a separate Answer Sheet. Please write only on the printed side of the sheet. Do not use the reverse side.*** If you have written something on any sheet which you do not want to be evaluated, cross it out.
- Use as many mathematical expressions as you think may help the evaluator to better understand your solutions. The evaluator may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).
- You are not allowed to leave your work desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please draw the attention of the proctor using the Help card.
- The beginning and end of the examination will be indicated by the proctor. Additionally, the proctor will announce the remaining time every hour and fifteen minutes before the end of the examination.
- At the end of the examination you must stop writing immediately. Sort your Answer Sheets according to Question numbering and put everything back in the envelope.
- Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the examination room.
- A list of constants and use-full relations are given on the next pages.

Fundamental constants

Speed of light in vacuum	c	=	2.998×10^{8} m s^{-1}
Planck constant	h	=	6.626×10^{-34} J s
Boltzmann constant	k_B	=	1.381×10^{-23} J K^{-1}
Stefan-Boltzmann constant	σ	=	5.670×10^{-8} W m^{-2} K^{-4}
Elementary charge	e	=	1.602×10^{-19} C
Universal gravitational constant	G	=	6.674×10^{-11} N m^2 kg^{-2}
Universal electric constant	ε_0	=	$8.854 \times 10^{-12} m^{-3} kg^{-1} s^4 A^2$
Universal gas constant	R	=	8.315 J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Wien's displacement law	$\lambda_m T$	=	2.898×10^{-3} m K
Mass of electron	m_e	=	9.109×10^{-31} kg
Mass of proton	m_p	=	1.673×10^{-27} kg
Mass of neutron	m_n	=	1.675×10^{-27} kg
Atomic mass unit (a.m.u.)		=	1.661×10^{-27} kg

Astronomical data

1 parsec	1 pc	=	3.086×10^{16} m
		=	206 265 au
		=	3.262 ly
1 astronomical unit (au)	1 au	=	1.496×10^{11} m
1 jansky	1 Jy	=	10^{-26} W m^{-2} Hz^{-1}
Hubble constant	H_0	=	70 km s^{-1} Mpc^{-1}
Solar luminosity	$L_\odot$	=	3.826×10^{26} W
Apparent angular diameter of Sun	$\theta_\odot$	=	32’
Effective temperature of Sun	$T_{\mathrm{eff},\odot}$	=	5778 K
Obliquity of the ecliptic (Earth)	ε	=	23.5°
Inclination of the lunar orbit w.r.t. ecliptic		=	05°08’43”
Apparent visual magnitude of full moon		=	–12.74

North Ecliptic Pole (J2000.0)	(α_E, δ_E)		($18^h00^m00^s$, +66°33'39")
North Galactic Pole (J2000.0)	(α_G, δ_G)		($12^h51^m26^s$, +27°07'42")
1 sidereal day		=	$23^h56^m04^s$
1 tropical year		=	365.2422 solar days
1 sidereal year		=	365.2564 solar days

Solar magnitudes

Apparent visual	=	–26.75
Absolute visual	=	+4.82
Apparent bolometric	=	–26.83
Absolute bolometric	=	+4.74

Solar system

Object	Mean radius (km)	Mass (kg)	Semi-major axis (au)	Eccentricity
Sun	695 500	1.988×10^{30}	---	---
Mercury	2 440	3.301×10^{23}	0.387	0.206
Venus	6 052	4.867×10^{24}	0.723	0.007
Earth	6 378	5.972×10^{24}	1.000 000	0.016 710
Moon	1 737	7.346×10^{22}	0.002 572	0.054 900
Mars	3 390	6.417×10^{23}	1.524	0.093
Jupiter	69 911	1.898×10^{27}	5.203	0.048
Saturn	58 232	5.683×10^{26}	9.537	0.054
Uranus	25 362	8.681×10^{25}	19.189	0.047
Neptune	24 622	1.024×10^{26}	30.070	0.009

Gauss's formulae

Cosine theorem : $cos\ a = cos\ b\ cos\ c + sin\ b\ sin\ c\ cos\ A$

Sine theorem : $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$

You might also need:

I. If the rate of change of a function 'f' is given by:

$$\frac{\Delta f}{\Delta t} = Be^{At}$$

Then the function will have a form:

$$f(t) = \frac{B}{A}(e^{At} - 1) + C$$

Where C is some constant dependent on the initial condition.

II. If the rate of change of a function 'f' is given by:

$$\frac{\Delta f}{\Delta t} = Af$$

Then the function will have a form:

$$f = Ce^{At}$$

Where C is some constant dependent on the initial condition.