13. Volcanic activity on Io, whose rotation period synchronizes with its orbital period, was proposed to be the result of tidal heating mainly from Jupiter. The resultant tidal force on a body is the difference in gravitational force experienced by the near and far sides of that body due to another body. Measurements of the surface distortion of Io via satellite radar altimeter mapping indicate that the surface rises and falls by up to 100m during one-half orbit. Only the surface layers will move by this amount. Interior layers within Io will move by a smaller amount, and thus we assume that on average the entire mass of Io is moved through 50m. Assume that Io is considered as two hemispheres each treated as a point mass. Calculate the *average power* of the tidal heating on Io.

Hint: you can use the following approximation $(1+x)^n \approx 1+nx$ for small *x*.

The mass of Io is $m_{Io} = 8.931938 \times 10^{22}\ kg$

The perijove distance is r_{peri} = 420000 km

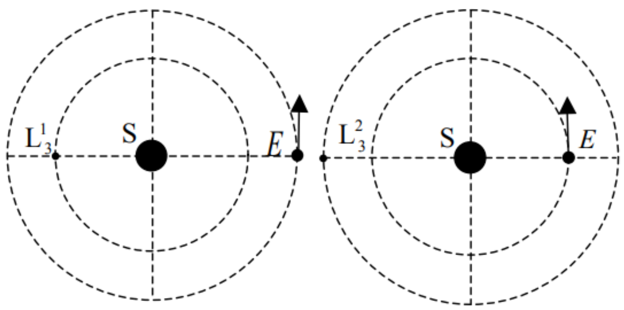
The apojove distance is r_{apo} = 423400 km

The orbital period of Io is 152853 s

The radius of Io is R_{Io} = 1821.6 km.

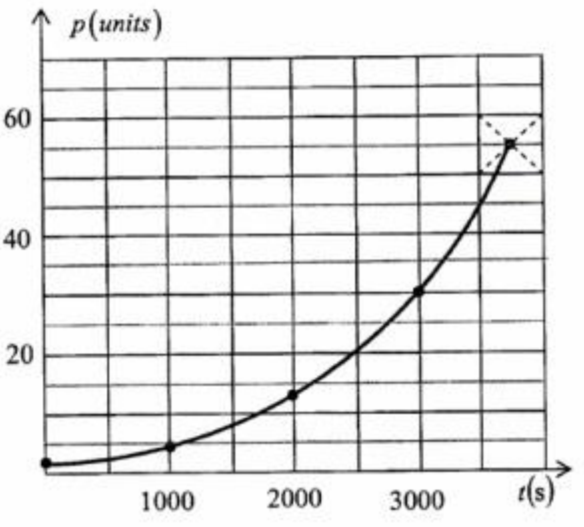
Problem 1. Lagrange Points

The *Lagrange* points are the five positions in an orbital configuration, where a small object is stationary relative to two big bodies, only gravitationally interacting with them. For example, an artificial satellite relative to Earth and Moon, or relative to Earth and Sun. In the **Figure 1** are sketched two possible orbits of Earth relative to Sun and of a small satellite relative to the Sun. Find out which of the two points L_3^1 and L_3^2 could be the real Lagrange point relative to the system Earth – Sun, and calculate its position relative to Sun. You know the following data: the Earth - Sun distance $d_{ES} = 15 \cdot 10^7$ km and the Earth – Sun mass ratio $M_E / M_S = 1/332946$



Problem 2Thermodynamic test

An hypotetical shuttle is launched to investigate the atmosphere (100% CO_2)of two extrasolar planets P_1 and P_2. The atmosphere is in static thermodynamic equilibrium. When the shuttle is near each planet, a radio probe is launched toward respective planet, invertical direction (in the direction of the planet`sradius). When the radio probe reaches constant velocity, it starts sending values of the pressure of the atmosphere. In Fig. 3.1 is plotted the atmospheric pressure values (in arbitrary units) as function of the time of descent for the planet P_1. When the probe touches the surface of planet P_1 it sends the value of the temperature $T_0 = 700\,\mathrm{K}$ and the value of the gravitational acceleration $g_0 = 10\,\mathrm{ms}^{-2}$.



The gravitational acceleration on each planet is assumed to be constant during uniform descent of the radio probes.

a) Find the altitude h_0 from where the radio probe R_1 starts the uniform descent and thus starts the transmitting information.

b) Find the temperature of planet P_1 at the altitude $h = 39.6\,\mathrm{km}$. You know: The universal gas constant $R = 8.3\,\mathrm{J/molK}$; the molar mass of CO_2, $\mu = 44\,\mathrm{g/mol}$.

c) In Fig. 3.2.was plotted the atmospheric pressure values (in arbitrary units) as a function of time of descent forthe planet P_2atmosphere. When the probe touched the surface of the planet P_2, it sends the value of the temperature $T_0 = 750\,\mathrm{K}$ and respectively the value of gravitational acceleration $g_0 = 8\,\mathrm{ms}^{-2}$

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the CO_2 atmosphere of the planet P_2.