Short Questions

Part	Solution	Mark
1	As a (rather) crude estimation, one may take $$\frac{dP}{dr}\sim\frac{\Delta P}{\Delta r}\sim\frac{-P}{R}=-\frac{GM_r\rho_r}{R^2}$$ We then obtain the scaling $$P=\frac{GM\rho}{R}\sim\rho^2R^2=A\rho^\beta$$ $$\rho=R^{\frac{2}{\beta-2}}$$ $$M\sim\rho R^3\sim R^{\frac{3\beta-4}{\beta-2}}$$ $$\alpha=\frac{3\beta-4}{\beta-2}$$	M1 M2 A1
2	$$H=\frac{\dot a}{a}=\frac{2}{3}\frac{1}{t}$$ The age of the universe could be obtained as $12.3+0.5\text{Gyr}=12.8\text{Gyr}$ $$\frac{2}{3}\frac{1}{12.8\text{Gyr}}=1.65\times10^{-18}s^{-1}=50.9\text{kms}^{-1}\text{Mpc}^{-1}$$ Setting k to 0 in the Friedmann equations, $$H^2=\frac{8\pi G}{3}\rho$$ $$\rho=\frac{3H^2}{8\pi G}=4.87\times10^{-27}\text{kgm}^{-3}$$	M2 M1 A1 M2 A1
3a	From the graph, we deduce the maximum angular distance to be $8.7'$, with a rotational velocity of $\sim240\ km/s$ (a value $\pm10\text{km/s}$ will be accepted as well) $$r=8.7'\times10.2\text{Mpc}=5.32\times10^9\text{ AU}$$ Under the general assumption of radial symmetry, $v(r)^2=\frac{GM(r)}{r}$ $$M=\frac{v^2r}{G}=6.88\times10^{41}\text{ kg}$$ $$[Accepted\ Range: 6.4\times10^{41}\text{ kg}-7.3\times10^{41}\text{ kg}]$$	M1 M1 M2 A1

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The power emitted is derived from the rate of axion mass decay into photons/energy

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = \frac{\ln 2}{\tau_{\frac{1}{2}}}$$

$$P_{dish} = \frac{\pi R_{dish}^2}{4\pi d^2} Mc^2\lambda = 2.7 \times 10^{-30} W$$

$$(\text{accepted range: } 2.5 \times 10^{-30} W - 2.9 \times 10^{-30} W)$$

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M1
M1
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<td>4a</td>
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We start by finding the angular radius of the Sun:

$$a_{sun} = \tan^{-1}\left(\frac{R_{sun}}{D}\right) \approx \frac{R_{sun}}{D} = 0.266^\circ$$

Where R_{sun} is the radius of the Sun and D is the distance to the Sun.

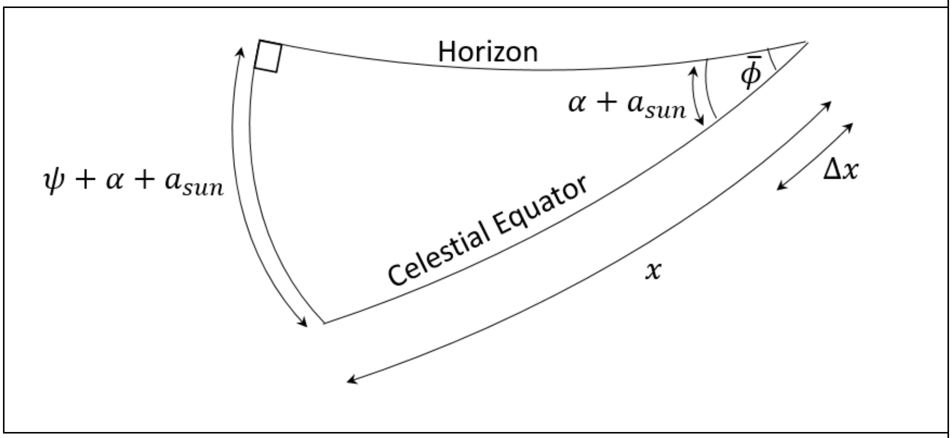
Next, we find the additional angle of depression the people at the top of the Burj Khalifa will have to see the new apparent horizon.

$$\psi = \cos^{-1}\left(\frac{R}{R+h}\right) = 0.924^\circ$$

Where R is the radius of the Earth and h is the height of the Burj Khalifa.

There is also a horizon depression due to atmospheric refraction: $\alpha = 32'$

Finally, we draw our spherical triangles. It is much simpler because this is the **Vernal Equinox**, so the Sun's path coincides with the Celestial Equator.



Using the labels in the diagram, we get the following:

$$\sin x = \frac{\sin(\psi + \alpha + a_{sun})}{\sin(\bar{\phi})}$$

$$\sin \Delta x = \frac{\sin(\alpha + a_{sun})}{\sin(\bar{\phi})}$$

The distance the sun has travelled across above the depressed horizon is $2x + 180^\circ$. Hence:

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M0.5
M2</td>
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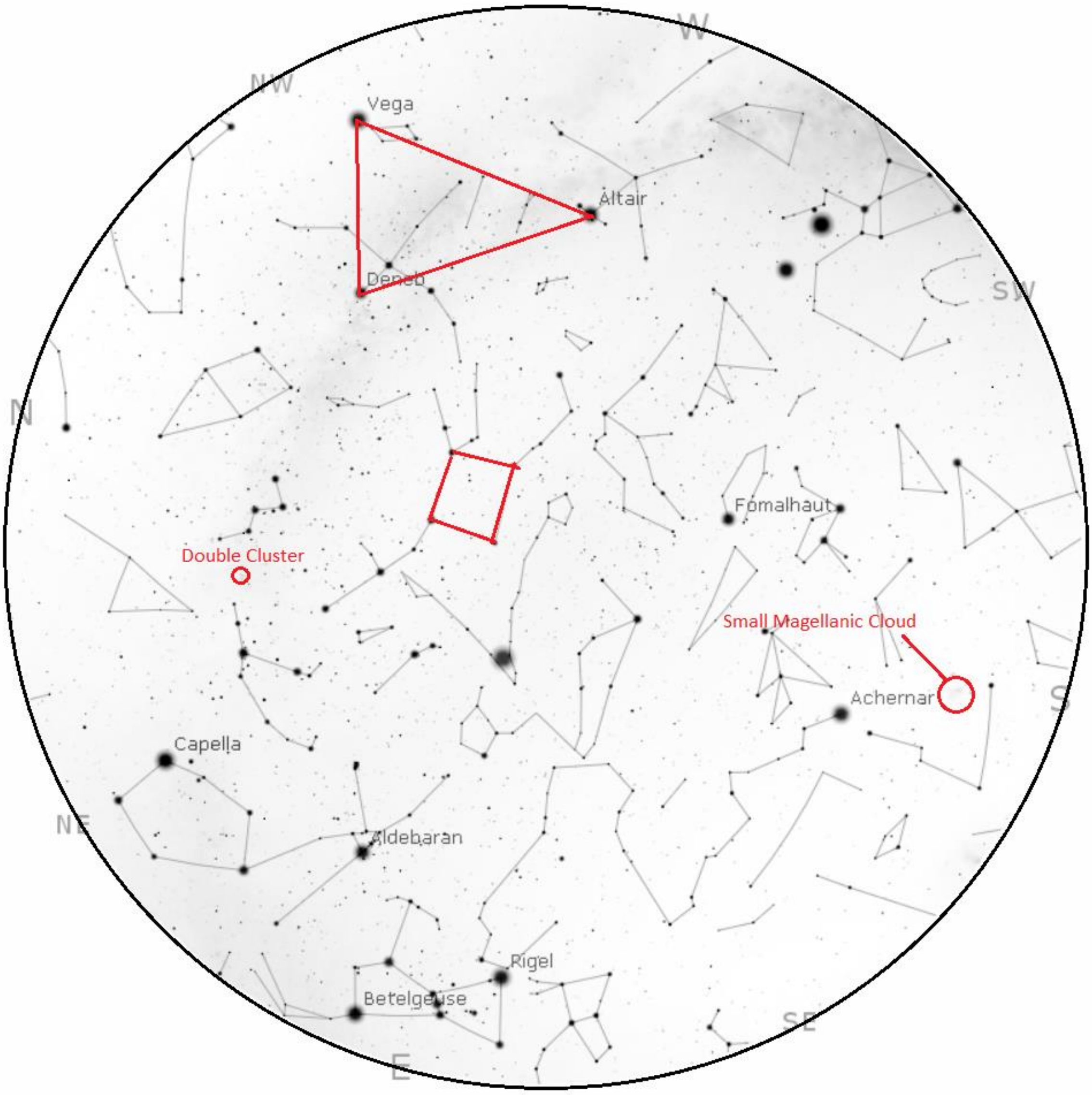
	$$T = \left(\frac{1^h}{15°}\right)(2x + 180°) = 12^h 15^m 14^s$$	M1 M1 A1
4b	The difference in sunset timing is due to the extra distance the Sun has to travel. In other words, $x - \Delta x$. Since this is symmetric for both sunrise **and** sunset, the extra fasting time is due to the sun travelling **double** that distance. $$\Delta T = 2^h \left(\frac{x - \Delta x}{15°}\right) = \left(\frac{2^h}{15°}\right)\left[\sin^{-1}\left(\frac{sin\,(\psi + \alpha + a_{sun})}{\cos\phi}\right) - \sin^{-1}\left(\frac{sin\,(\alpha + a_{sun})}{cos\,\phi}\right)\right]$$ Plugging in the values, we get: $$\Delta T = 8^m 10^s$$	
5	1. See chart below 2. See chart below 3. See chart below for common names. Bayer designations also accepted: Alpha Aurigae, Alpha Eridani, Alpha Piscis Austrini (deduct half point if spelt incorrectly, for eg. Eridanus instead of Eridani) 4. See chart below	A1 A2 A2 x 3 A2 x 2


W
NW
SW
N
S
NE
SE
E
Vega
Altair
Deneb
Fomalhaut
Double Cluster
Small Magellanic Cloud
Achernar
Capella
Aldebaran
Rigel
Betelgeuse