1 Stars

Suppose we have an equation of state for stars:

$$P = A\rho^{\beta} \tag{1}$$

where P is the pressure, ρ is the density, and A is a constant. Such a solution is called a *polytrope*, and one can **estimate** an approximate relationship between the mass M and density ρ of the form $M \propto R^{\alpha}$.

Find α. State any assumptions you made. [4]

Hint: The hydrostatic equilibrium condition given by $\frac{dP}{dr} = -\frac{GM_r\rho_r}{r^2}$ *may be useful.*

2 Cosmology

In a flat, **matter-dominated** universe, the scale factor a is described by the equation:

$$a = \left(\frac{t}{t_0}\right)^{2/3} \tag{2}$$

where t_0 is the present age of the universe, at which $a = 1$. A spectroscopic study of uranium/thorium content revealed the age of the galaxy to be 12.3 Gyr. Given that the galaxy was formed 500 million years after the Big Bang, what value of the Hubble Constant H_0 would be consistent with this estimate? [4]
Determine the corresponding value of the critical density as well. [3]

If you may require it, the Friedmann equations are given by:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{3}$$

You are also given that $\frac{d}{dx}(x^n) = nx^{n-1}$.

3 Galaxies

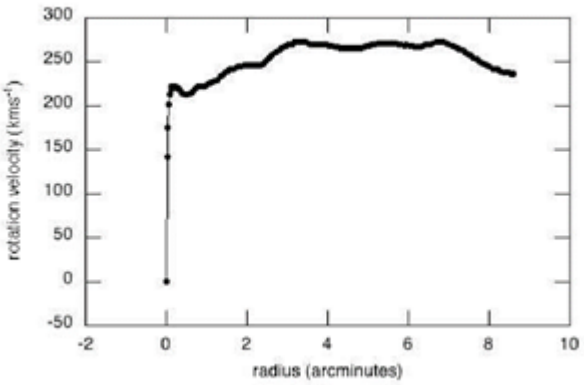

Figure 1: NGC4565

Figure 1 shows the measured rotation velocity of hydrogen gas around the centre of galaxy NGC4565 as a function of angular separation from the galaxy. NGC4565 is located 10.2 MPc away from Earth.

1. Estimate the total mass within the galaxy within the largest radius for which the data is available. [5]

2. Assume that the mass of galaxy is dominated by *axions* of rest energy $50\,\mu$eV having a lifetime of decay into two photons of 10^{40} seconds. Calculate the power from photons incident on a 100 cm diameter radio dish here due to axion decay. [4]

4 Long Fast

The tallest building in the world is the Burj Khalifa at 828 m tall. During the holy month of Ramadan, observing Muslims will fast from the time between Sunrise and Sunset. Due to the height of the Burj Khalifa, people at the top will fast for a longer period than people at the bottom.

For simplicity, suppose it is March 21 2025. Calculate the time that a person at the top of Burj Khalifa will fast for on this day. [6]
What is the extra amount of time (to the nearest second) that a person at the top of Burj Khalifa will fast for, as compared to a person on the ground? [3]

Burj Khalifa Coordinates: 25.1972° N, 55.2744° E.
You are to account for atmospheric refraction with a value of 32'.
Hint: The Sun is ***not*** *a point source.*

5 Practical

The star chart below shows the night sky at 0000h local time on Sep 22, 2020 from Bogota, Colombia (UTC -5:00), which is at latitude 4.7110°N, 74.0721°W. Complete the questions on the following page, and write your answers on the star chart itself. Note that the size of stars and objects are scaled by their brightness in the night sky, with brighter objects appearing larger.

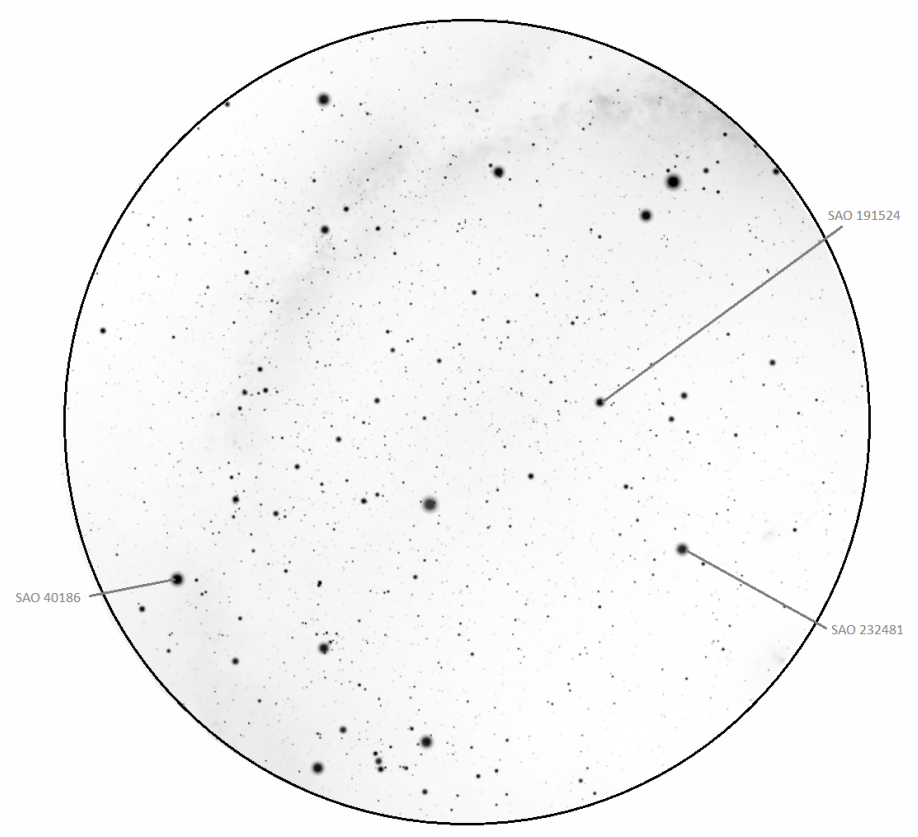

Figure 2: Starchart

1. As a warm up, first label the cardinal directions (North South East West) on the boundary of the star chart (ie. Horizon). [1]

2. Find the Summer Triangle (consisting of Deneb, Altair and Vega) and connect the three stars on the chart. [2]

3. A few stars have been marked according to the SAO catalogue. Identify these objects by either their common names or their Bayer designations (for example, Aldebaran or α Tauri), and write your answers next to the respective SAO labels in the figure. [6]

4. Find the Perseus Double Cluster, and the Small Magellanic Cloud in Figure 2 and mark each of them with a small hollow circle. [4]